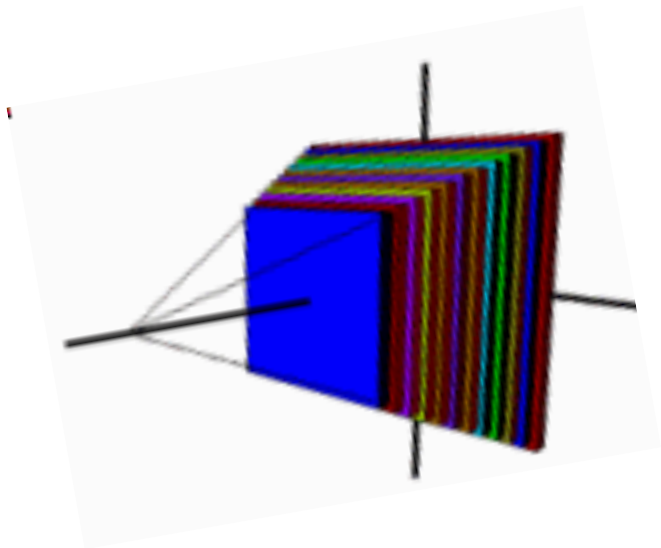
Volumes of Solids with Known Cross Sections

In this section, we are going to look at cross sections that are not circular.

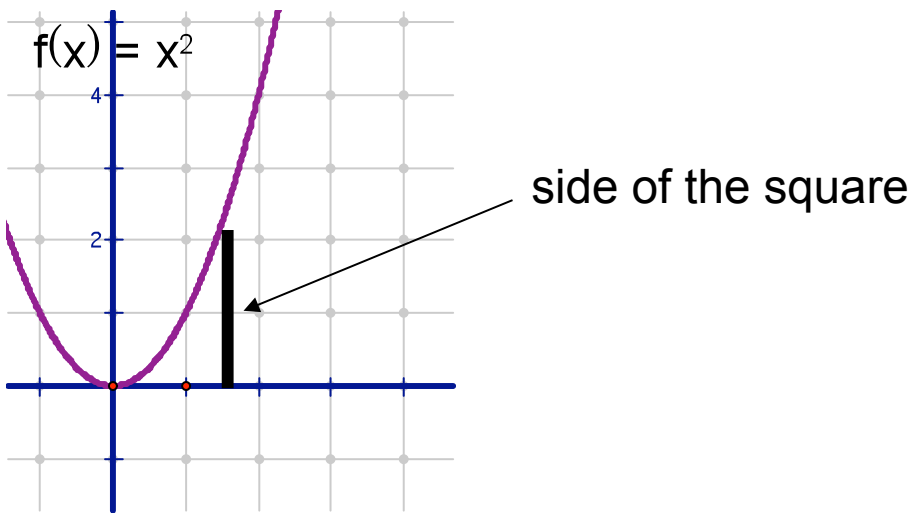


Our figures will be defined by two functions. In general, we use squares or equilateral triangles, because we only need one side to calculate the area:

$$A_{\text{square}} = s^2$$

$$A_{\text{equilateral triangle}} = \frac{\sqrt{3}}{4} s^2$$

Example: Find the volume of the solid whose cross section is a square with the side defined by the distance between $y = x^2$ and the x-axis, from 0 to 2.



The area of the square will be $A = (x^2)^2 = x^4$. To accumulate this area, we will use the following integral. (Notice we do not need π anymore, since it is a square, not a circle.)

$$V = \int_0^2 x^4 dx$$

$$= \frac{1}{5} x^5 \Big]_0^2$$

$$= \frac{32}{5}$$