Differentiating Inverse Trig Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Differentiate each of the following. The answers are provided on the next page.

1. $\frac{d}{dx}[\arcsin x^2]$

2. $\frac{d}{dx}[\arccos 3x]$

3. $\frac{d}{dx}\left[\operatorname{arcsec}\sqrt{x}\right]$

4. $\frac{d}{dx}\left[\arctan e^x\right]$

Answers

1. $$\frac{d}{dx}[\arcsin x^2] = \frac{2x}{\sqrt{1-(x^2)^2}} = \frac{2x}{\sqrt{1-x^4}}$$

2. $$\frac{d}{dx}[\arccos 3x] = \frac{-3}{\sqrt{1-(3x)^2}} = \frac{-3}{\sqrt{1-9x^2}}$$

3. $$\frac{d}{dx}\left[\operatorname{arcsec}\sqrt{x}\right] = \frac{\frac{1}{2\sqrt{x}}}{|\sqrt{x}|\sqrt{\left(\sqrt{x}\right)^2-1}} = \frac{\frac{1}{2\sqrt{x}}}{|\sqrt{x}|\sqrt{x-1}}$$

$$= \frac{1}{2x\sqrt{x-1}}$$

4. $$\frac{d}{dx}\left[\arctan e^x\right] = \frac{e^x}{1+(e^x)^2} = \frac{e^x}{1+e^{2x}}$$