Inverse Trigonometric Functions

In this lesson, we are going to review inverse trig functions. We are going to graph inverse trig functions, evaluate inverse trig expressions, and solve equations involving inverse trig functions.

Recall that there are two ways to write inverse trig functions:

1. Inverse sine is written as $\arcsin x$ or $\sin^{-1} x$.
2. Inverse cosine is written as $\arccos x$ or $\cos^{-1} x$.
3. Inverse tangent is written as $\arctan x$ or $\tan^{-1} x$.

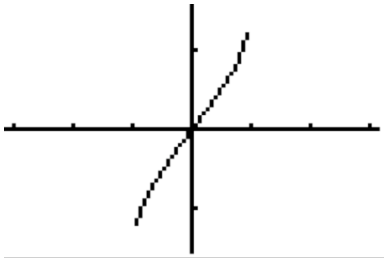
Here are the domains and ranges of each inverse trig function.

$y = \arcsin x$	$y = \arccos x$	$y = \arctan x$
$D = [-1, 1]$	$D = [-1, 1]$	$D = (-\infty, \infty)$
$R = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$R = [0, \pi]$	$R = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

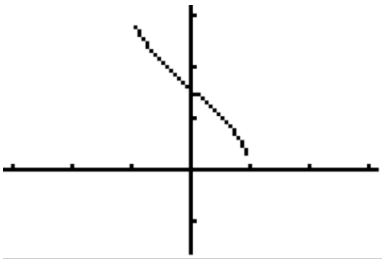
$y = \operatorname{arccot} x$	$y = \operatorname{arcsec} x$	$y = \operatorname{arccsc} x$
$D = (-\infty, \infty)$	$D = (-\infty, -1] \cup [1, \infty)$	$D = (-\infty, -1] \cup [1, \infty)$
$R = (0, \pi)$	$R = \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$	$R = \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

Here are the graphs:

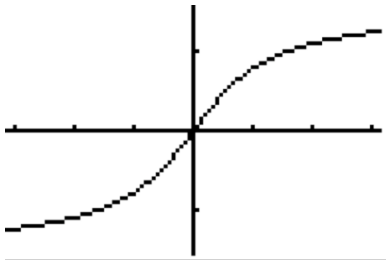
$y = \arcsin x$



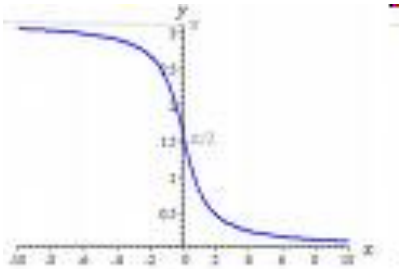
$y = \arccos x$



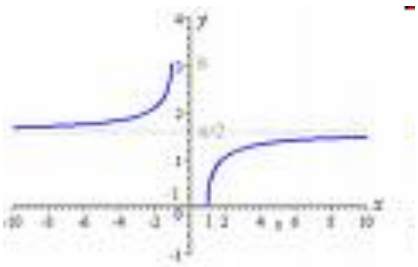
$y = \arctan x$



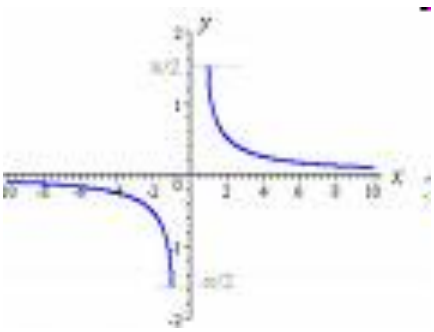
$y = \operatorname{arccot} x$



$y = \operatorname{arcsec} x$



$y = \operatorname{arccsc} x$



Example 1: Evaluate the following.

a. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

c. $\arctan 1$

d. $\arccos(0.7)$

Answers:

a. $\frac{\pi}{3}$

b. $\frac{5\pi}{6}$

c. $\frac{\pi}{4}$

d. 0.795 (from calculator)

Example 2: Solve the following equation.

$$\arcsin(x-5) = \frac{\pi}{3}$$

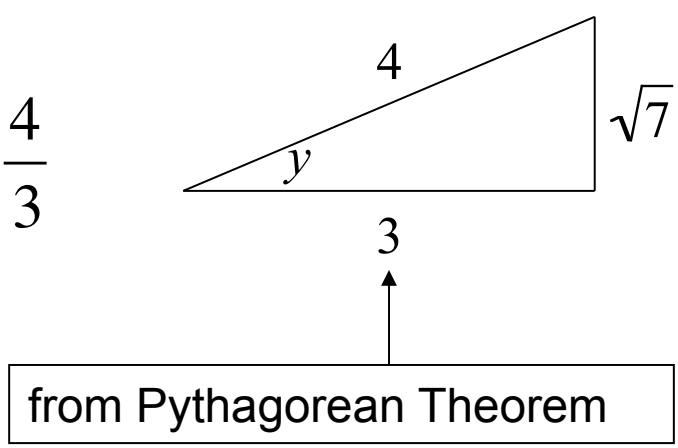
$$\sin[\arcsin(x-5)] = \sin\frac{\pi}{3}$$

$$x-5 = \frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3}}{2}+5$$

Example 3: Find $\sec y$ given the following:

$$y = \arcsin\left(\frac{\sqrt{7}}{4}\right)$$

We use a right triangle to solve this one, with our old trig definitions:

$$\sec y = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{4}{3}$$