Review Chapter 4
Part 1: Calculator Allowed
Answers

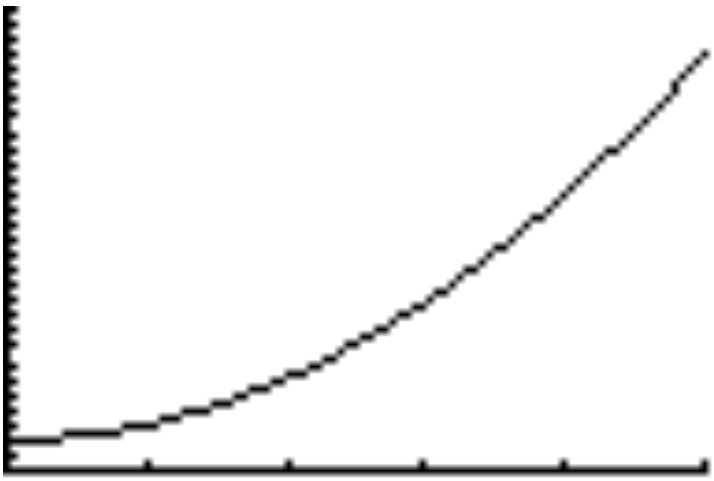
1. Be able to evaluate definite integrals on your calculator. The required functions on your calculator are most likely the following:

   MATH 9

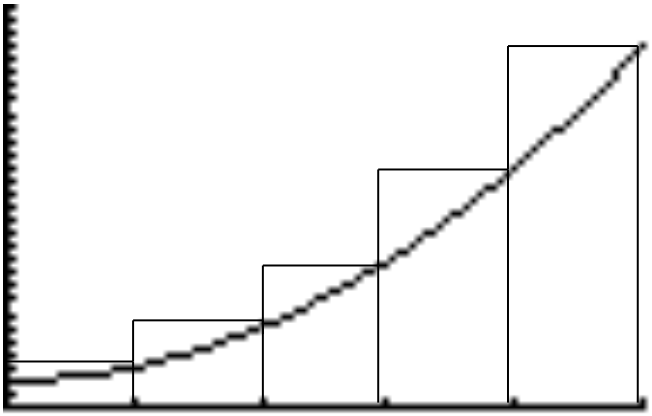
   fnInt(f(x),x,lower,upper)

2. Calculate the upper, lower, and midpoint Riemann sums of the following function:

   $f(x) = x^2 + 2$ on $[0, 5]$ with $n = 5$



Upper

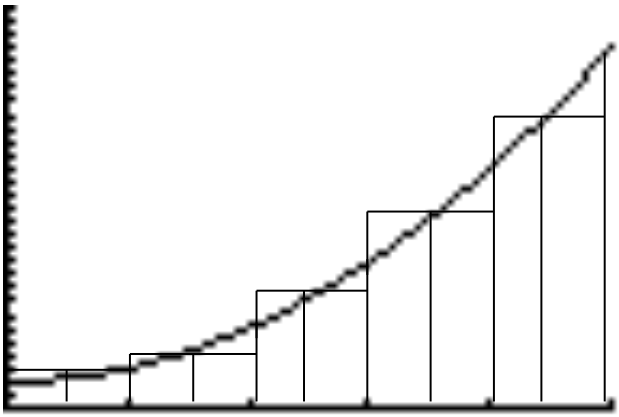


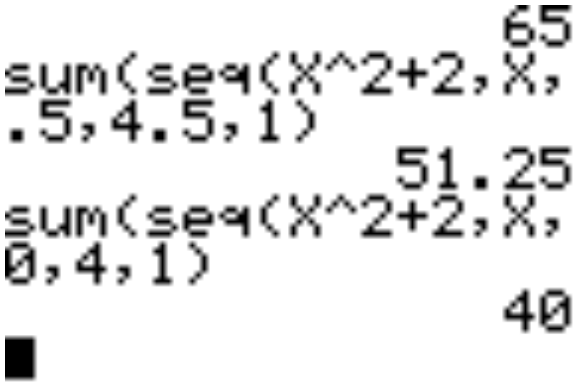
$$U = 1[f(1) + f(2) + f(3) + f(4) + f(5)]$$
$$U = 1(3 + 6 + 11 + 18 + 27)$$
$$U = \boxed{65}$$

Lower

$$L = 1[f(0) + f(1) + f(2) + f(3) + f(4)]$$
$$L = 1(2 + 3 + 6 + 11 + 18)$$
$$L = \boxed{40}$$

Midpoint





$$M = 1\left[f\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right) + f\left(\frac{7}{2}\right) + f\left(\frac{9}{2}\right)\right]$$
$$M = 1\left(\frac{9}{4} + \frac{17}{4} + \frac{33}{4} + \frac{57}{4} + \frac{89}{4}\right)$$
$$M = \frac{205}{4} = \boxed{51.25}$$

3. Use the Trapezoidal Rule to approximate the following:

$$\int_0^2 x^3 dx, \qquad n = 4$$

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

what we "count by" from 0 to 2

$$\boxed{\frac{b-a}{2n}}$$

$$\frac{2-0}{2\cdot 4}\left[f(0)+2f\left(\frac{1}{2}\right)+2f(1)+2f\left(\frac{3}{2}\right)+f(2)\right]$$

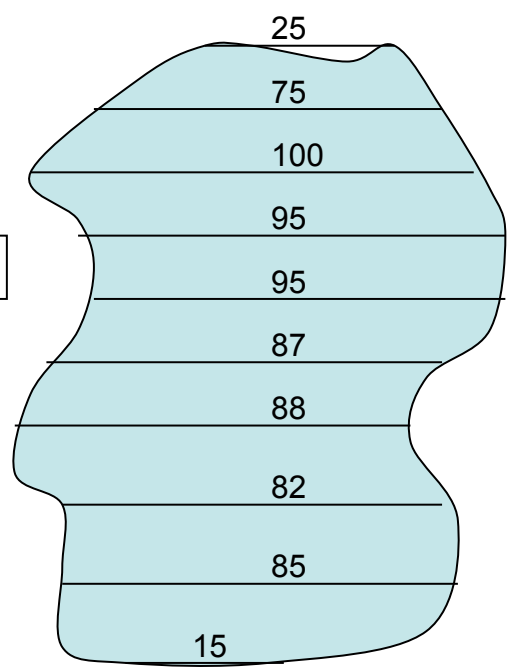
$$\frac{1}{4}\left(2\cdot\frac{1}{8}+2\cdot 1+2\cdot\frac{27}{8}+8\right)=4.25$$

4. Approximate the area of the lake using the Trapezoidal Rule.

$$\frac{1}{2}\cdot 15\left(25+2\cdot 75+2\cdot 100+2\cdot 95+2\cdot 95+2\cdot 87+2\cdot 88+2\cdot 82+2\cdot 85+15\right)$$

$$=10{,}905$$

$$\boxed{\Delta x = 15}$$



5. Find the area between the given function and the x-axis from -2 to 5.

$$f(x) = x^2 - 2x - 3$$

$$\int_{-2}^{-1}(x^2 - 2x - 3)dx - \int_{-1}^{3}(x^2 - 2x - 3)dx + \int_{3}^{5}(x^2 - 2x - 3)dx$$

$$23.666$$

6. Find the same area using absolute value.

$$\int_{-2}^{5}\left|x^2 - 2x - 3\right|dx$$

$$23.666$$

7. A particle moves along the x-axis with the following velocity from 0 to 6 seconds. The particle is in position 4 at time 0. Find the position function, displacement, and total distance traveled.

$$v(t) = t^2 - 7t + 10 \text{ on } 0 \le t \le 6$$

$$s(t) = \int (t^2 - 7t + 10)\,dt$$

$$s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + C \quad [\text{use } s(0) = 4]$$

$$C = 4$$

$$\boxed{s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 4}$$

$$\boxed{\int_0^6 v(t)\,dt = 6} \longleftarrow \text{displacement}$$

$$\boxed{\int_0^6 |v(t)|\,dt = 15} \longleftarrow \text{total distance traveled}$$

8. Know how to solve application problems involving average value and accumulation. Here's a list of one's you've already done for you to review.

   p. 284, exercise 33

   p. 285, exercise 63

   p. 299, exercises 105 and 106

Review Chapter 4
Part 2: No Calculator
Answers

9. Write the definition of definite integral.

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)\,dx$$

10. Write the Fundamental Theorem of Calculus.

If $F(x)$ is an antiderivative of $f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

11. Write the Second Fundamental Theorem of Calculus.

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

For questions 12−23, find the integrals.

12. $\int \frac{2}{x^3}\,dx$

$\int 2x^{-3}\,dx$

$-\frac{1}{2}\cdot 2x^{-2}+C$

$$\boxed{\begin{array}{c} -x^{-2}+C \\ -\frac{1}{x^2}+C \end{array}}$$

13. $\int \sqrt{x}(x+2)\,dx$

$\int x^{3/2}+2x^{1/2}\,dx$

$\frac{2}{5}x^{5/2}+\frac{2}{3}\cdot 2x^{3/2}+C$

$$\boxed{\frac{2}{5}x^{5/2}+\frac{4}{3}x^{3/2}+C}$$

14. $\int -2\sec x\tan x\,dx$

$$\boxed{-2\sec x+C}$$

15. $\int \frac{x^2+2x}{x^4}\,dx$

$\int \left(\frac{x^2}{x^4}+\frac{2x}{x^4}\right)dx$

$\int \left(\frac{1}{x^2}+\frac{2}{x^3}\right)dx$

$\int (x^{-2}+2x^{-3})\,dx$

$-x^{-1}-\frac{1}{2}\cdot 2x^{-2}+C$

$$\boxed{-\frac{1}{x}-\frac{1}{x^2}+C}$$

16. $\int_1^3 (3x^2+5x-4)\,dx$

$$\left[x^3+\frac{5}{2}x^2-4x\right]_1^3$$

$$27+\frac{45}{2}-12-\left(1+\frac{5}{2}-4\right)$$

$$\frac{75}{2}-\left(-\frac{1}{2}\right)$$

$$\boxed{38}$$

17. $\int_{\pi/4}^{3\pi/4} \sin x\,dx$

$$-\cos x\Big]_{\pi/4}^{3\pi/4}$$

$$-\cos\left(\frac{3\pi}{4}\right)-\left(-\cos\left(\frac{\pi}{4}\right)\right)$$

$$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}$$

$$\boxed{\sqrt{2}}$$

18. $\int 4x^3(x^4+2)^2\,dx$

Let $u = x^4+2$.

$$du = 4x^3dx$$

$$\int u^2du = \frac{1}{3}u^3+C$$

$$\boxed{\frac{1}{3}(x^4+2)^3+C}$$

19. $\int \frac{x^2}{\sqrt{1+x^3}}\,dx$

Let $u = 1+x^3$.

$$du = 3x^2dx$$

$$\frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{3}\int u^{-1/2}du$$

$$= \frac{1}{3}\cdot\frac{2}{1}u^{1/2}+C$$

$$= \boxed{\frac{2}{3}\sqrt{1+x^3}+C}$$

20. $\int \tan^2 x \sec^2 x \, dx$

Let $u = \tan x$.

$du = \sec^2 x \, dx$

$\int u^2 du = \frac{1}{3} u^3 + C$

$= \boxed{\frac{1}{3} \tan^3 x + C}$

21. $\int x\sqrt{x+2} \, dx$

Let $u = x + 2$. $\quad x = u - 2$

$du = dx$

$\int (u-2)\sqrt{u} \, du$

$\int u^{3/2} - 2u^{1/2} du$

$\frac{2}{5} u^{5/2} - 2 \cdot \frac{2}{3} u^{3/2} + C$

$\boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$

22. $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$

Let $u = 2x + 1$.

$du = 2dx$

$x = 4 \Rightarrow u = 9$

$x = 0 \Rightarrow u = 1$

$\frac{1}{2} \int_1^9 u^{-1/2} du$

$\frac{1}{2} \cdot 2u^{1/2} \Big]_1^9$

$3 - 1$

$\boxed{2}$

23. $\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right) dx$

Let $u = \frac{2x}{3}$.

$du = \frac{2}{3} dx$ $\left(\text{need } \frac{3}{2} \text{ to balance}\right)$

$x = \frac{\pi}{2} \Rightarrow u = \frac{\pi}{3}$

$x = 0 \Rightarrow u = 0$

$\frac{3}{2} \int_0^{\pi/3} \cos u \, du = \frac{3}{2} \cdot \sin u \Big]_0^{\pi/3}$

$\frac{3}{2}\left(\sin\frac{\pi}{3} - \sin 0\right) = \boxed{\frac{3\sqrt{3}}{4}}$

24. Solve the following differential equation:

$$\frac{dy}{dx} = \frac{1}{2}x + 2 \text{ with initial condition } y = 2 \text{ when } x = 4.$$

$$dy = \left(\frac{1}{2}x + 2\right)dx$$

$$\int dy = \int \left(\frac{1}{2}x + 2\right)dx$$

$$y = \frac{1}{4}x^2 + 2x + C \quad (\text{use } y = 2 \text{ when } x = 4)$$

$$2 = \frac{1}{4} \cdot 16 + 2 \cdot 4 + C$$

$$2 = 4 + 8 + C$$

$$C = -10$$

$$\boxed{y = \frac{1}{4}x^2 + 2x - 10}$$

25. Be able to solve “no work” integrals like on page 273 (exercises 33−45) and page 285 (exercises 54−60).

26. Solve using the Second Fundamental Theorem of Calculus.

a. If $F(x) = \int_3^{x^3} \sqrt{t}\,dt$, find $F'(x)$.

$$F'(x) = \sqrt{x^3} \cdot 3x^2$$

$$F'(x) = \boxed{3x^3\sqrt{x}}$$

b. If $F(x) = \int_{-2}^{\cos x} 3t^2 dt$, find $F'(x)$.

$$F'(x) = 3\cos^2 x \cdot -\sin x$$

$$F'(x) = \boxed{-3\sin x \cos^2 x}$$

27. Find the average value of the following function on the given interval and the x-value where it occurs.

$$f(x) = 3x^2 - 2x \text{ on } [1,4]$$

$$\frac{1}{4-1}\int_1^4 (3x^2 - 2x)dx = \frac{1}{3}\left[x^3 - x^2\right]_1^4$$

$$= \frac{1}{3}\left[(4^3 - 4^2) - (1^3 - 1^2)\right]$$

$$= \frac{1}{3}(48)$$

$$= \boxed{16}$$

$$3x^2 - 2x = 16$$

$$3x^2 - 2x - 16 = 0$$

$$(3x - 8)(x + 2) = 0$$

$$\boxed{x = \frac{8}{3}} \qquad \cancel{x = -2} \longleftarrow \text{Not between 1 and 4}$$