Answers to Inverse Trig Functions Review

Find each of the following.

1. $\arccos 1 = 0$

2. $\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

3. $\sin\left(\arccos \frac{5}{13}\right) = \frac{12}{13}$

4. $\tan\left(\arcsin \frac{\sqrt{2}}{2}\right) = 1$

5. $\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$

6. $\frac{d}{dx}[\operatorname{arccsc} x] = \frac{-1}{|x|\sqrt{x^2-1}}$

7. $\frac{d}{dx}[\operatorname{arccot} x] = \frac{-1}{1+x^2}$

8. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin \frac{x}{a} + C$

9. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a}+C$

10. $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\operatorname{arcsec}\frac{|x|}{a}+C$

11. $\frac{d}{dx}[\arcsin(9x)] = \frac{9}{\sqrt{1-81x^2}}$

12. $\frac{d}{dx}[\operatorname{arcsec}(x^2)] = \frac{2x}{|x^2|\sqrt{x^4-1}}$

13. $\frac{d}{dx}\left[\operatorname{arccot}(\sqrt{x})\right] = \frac{\frac{-1}{2\sqrt{x}}}{1+x} = \frac{-1}{2\sqrt{x}(1+x)}$

14. $\int \frac{1}{9+x^2}\,dx = \frac{1}{3}\arctan\frac{x}{3}+C$

15. $\int \frac{1}{x\sqrt{9x^2-1}}\,dx = \operatorname{arcsec}|3x|+C$

16. $\int_0^{1/4} \frac{1}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\int_0^{1/2} \frac{1}{\sqrt{1-u^2}}\,du = \frac{1}{2}\arcsin u\Big]_0^{1/2} = \frac{1}{2}\left(\arcsin\frac{1}{2} - \arcsin 0\right) = \frac{\pi}{12}$

17. $\int_3^5 \frac{1}{x^2-4x+13}\,dx$

$$\begin{aligned}
\int_3^5 \frac{1}{x^2-4x+13}\,dx &= \int_3^5 \frac{1}{x^2-4x+4+9}\,dx \\
&= \int_3^5 \frac{1}{(x-2)^2+3^2}\,dx \\
&= \frac{1}{3}\arctan\frac{x-2}{3}\Big]_3^5 \\
&= \frac{1}{3}\left(\arctan 1 - \arctan\frac{1}{3}\right) \\
&\approx 0.155
\end{aligned}$$