Answers to Growth and Decay Problems

1. $$y = Ce^{kt}$$

If 1990 is $t = 0$, then $C = 5{,}300{,}000{,}000$.

$$y = 5{,}300{,}000{,}000e^{kt}$$

Now we have to find k. The year 2050 means $t = 60$.

$$10{,}000{,}000{,}000 = 5{,}300{,}000{,}000e^{60k}$$

$$\frac{100}{53} = e^{60k}$$

$$\ln\frac{100}{53} = 60k$$

$$k = \frac{\ln\frac{100}{53}}{60} \approx 0.01058$$

$$\therefore\ y = 5{,}300{,}000{,}000e^{\frac{\ln\frac{100}{53}}{60}t}$$

Now we will solve for t when $y = 18{,}000{,}000{,}000$.

$$18{,}000{,}000{,}000 = 5{,}300{,}000{,}000e^{\frac{\ln\frac{100}{53}}{60}t}$$

$$\frac{180}{53} = e^{\frac{\ln\frac{100}{53}}{60}t}$$

$$\ln\frac{180}{53} = \frac{\ln\frac{100}{53}}{60}t$$

$$t \approx 115.550 \text{ years}$$

About 116 years, which is the year 2106.

2. $$y = Ce^{kt}$$

We will use C as the initial amount and have ½ C left after 8.14 days.

$$\frac{1}{2}C = Ce^{k(8.14)}$$

$$\frac{1}{2} = e^{8.14k}$$

$$\ln\frac{1}{2} = 8.14k$$

$$k = \frac{\ln\frac{1}{2}}{8.14}$$

$$\therefore\ y = Ce^{\frac{\ln\frac{1}{2}}{8.14}}$$

We want to find the value of C that will result in 100 mg after two days.

$$100 = Ce^{\frac{\ln\frac{1}{2}}{8.14}t}$$

$$\boxed{C \approx 118.567 \text{ mg}}$$

3.

$$y = Ce^{kt}$$

$$C = 1{,}000{,}000$$

$$\therefore\ y = 1{,}000{,}000e^{kt}$$

It doubles in two days.

$$2{,}000{,}000 = 1{,}000{,}000e^{k(2)}$$

$$2 = e^{2k}$$

$$\ln 2 = 2k$$

$$k = \frac{\ln 2}{2}$$

$$\therefore\ y = 1{,}000{,}000e^{\frac{\ln 2}{2}t}$$

Since one week is seven days, $t = 7$.

$$y = 1{,}000{,}000e^{\frac{\ln 2}{2}(7)}$$

$$y \approx 11{,}313{,}708.499$$

$\boxed{\text{There will be 11,313,708 bacteria in one week.}}$

4. $$y = Ce^{kt}$$

We need to find k. In 1590 years we have half of what we started with.

$$\frac{1}{2}C = Ce^{k(1590)}$$

$$\frac{1}{2} = e^{1590k}$$

$$\ln\frac{1}{2} = 1590k$$

$$k = \frac{\ln\frac{1}{2}}{1590} \approx -0.0004359$$

$$\therefore y = Ce^{\frac{\ln\frac{1}{2}}{1590}t}$$

The problem gives us $C = 95$ g.

$$\therefore y = 95e^{\frac{\ln\frac{1}{2}}{1590}t}$$

a. In 20 years, $y = 95e^{\frac{\ln\frac{1}{2}}{1590}(20)} \approx 94.175$ g.

b. In 1000 years, $y = 95e^{\frac{\ln\frac{1}{2}}{1590}(1000)} \approx 61.432$ g.

5. $$y = Ce^{kt}$$

We need to find k. We are given $C = 500$ and $y = 8000$ when $t = 3$.

$$8000 = 500e^{k(3)}$$

$$16 = e^{3k}$$

$$\ln 16 = 3k$$

$$k = \frac{\ln 16}{3}$$

$$\therefore y = 500e^{\frac{\ln 16}{3}t}$$

a. In 4 hours,

$$y = 500e^{\frac{\ln 16}{3}(4)} \approx 20{,}158.737$$

$$\boxed{\text{approximately 20,159 bacteria}}$$

b. How long will it take to reach 30,000 bacteria?

$$30{,}000 = 500e^{\frac{\ln 16}{3}t}$$

$$60 = e^{\frac{\ln 16}{3}t}$$

$$\ln 60 = \frac{\ln 16}{3}t$$

$$t = \frac{3\ln 60}{\ln 16} \approx 4.430$$

$$\boxed{\text{about 4.430 hours}}$$

6. $$y = Ce^{kt}$$

If the dog weighs 10 kg, we will need (10)(20) = 200 mg of sodium pentobarbital at the end of the half hour. We need to find k.

$$\frac{1}{2}C = Ce^{k(5)}$$

$$\frac{1}{2} = e^{5k}$$

$$\ln\frac{1}{2} = 5k$$

$$k = \frac{\ln\frac{1}{2}}{5}$$

$$\therefore\ y = Ce^{\frac{\ln\frac{1}{2}}{5}t}$$

We want 200 mg after a half hour.

$$200 = Ce^{\frac{\ln\frac{1}{2}}{5}\left(\frac{1}{2}\right)} \approx 214.345$$

$$\boxed{\text{approximately 214.345 mg}}$$