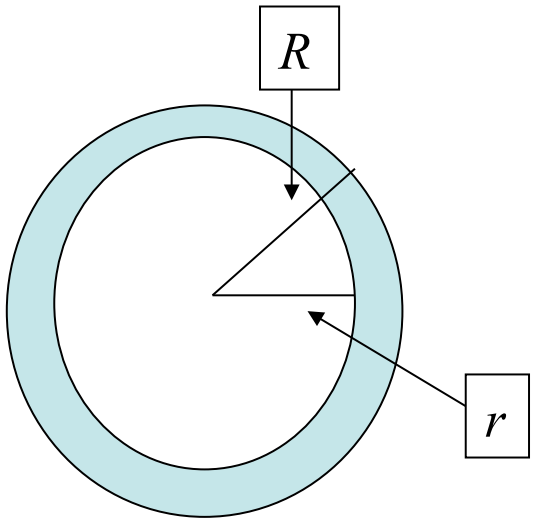
Volumes of Revolution: The Washer Method

The disk method can be extended to cover solids of revolution with holes in them by using a washer.

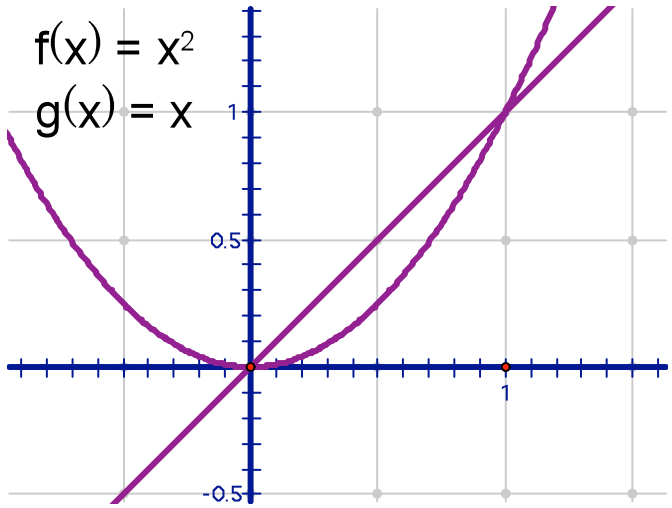


The area of the washer is the area of the big circle minus the area of the little circle. This difference is the area we are going to accumulate.

The Washer Method

$$V = \pi \int_a^b [R(x)^2 - r(x)^2]dx$$

Example: Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x$ and $y = x^2$ about the x-axis.



$$\boxed{\begin{aligned} x^2 &= x \\ x^2 - x &= 0 \\ x(x-1) &= 0 \\ x = 0 \quad & x = 1 \end{aligned}}$$

$$\begin{aligned} V &= \pi \int_0^1 \left[x^2 - (x^2)^2 \right] dx \\ &= \pi \int_0^1 (x^2 - x^4)\, dx \\ &= \pi \left[\frac{1}{3} x^3 - \frac{1}{5} x^5 \right]_0^1 \\ &= \pi \left[\left(\frac{1}{3} - \frac{1}{5} \right) - 0 \right] \\ &= \frac{2}{15} \pi \end{aligned}$$