Differential Equations

$$\frac{dy}{dx} = 3x^2 + 1$$

Solve for y by integrating both sides.

$$dy = (3x^2 + 1)dx$$

$$\int dy = \int (3x^2 + 1)dx$$

$$y = x^3 + x + C$$

First, we separate the variables, then integrate.

This represents the family of curves whose derivative would be $3x^2 + 1$.

Now, we can find the C value if we know some initial condition. Let's suppose y passes through $(2, 4)$.

$$y = x^3 + x + C$$

$$4 = 2^3 + 2 + C$$

$$4 = 10 + C$$

$$-6 = C$$

So, given the initial condition, the equation would be

$$y = x^3 + x - 6$$

Now it is a specific curve, instead of a family of curves.

Let's look at an example.

Find the equation for y if $\frac{dy}{dx} = 2x - 1$ and y passes through (1, 1).

$$dy = (2x - 1)dx$$ Separate the variables,

$$\int dy = \int (2x - 1)dx$$ integrate,

$$y = x^2 - x + C$$ plug in the initial condition.

At (1, 1),

$$1 = 1^2 - 1 + C$$

$$1 = C$$

So, the function is $y = x^2 - x + 1$.

Now, let's look at a position, velocity, and acceleration problem.

A ball is thrown upward at $26\ \mathrm{m/s}$ from a platform $15\ \mathrm{m}$ above the ground. Find, using integration, the position function.

If we consider the position function, the velocity function, and the acceleration function, we can make a relationship using integration. Here is what we know:

position:	$s(t)$
velocity:	$s'(t) = v(t)$
acceleration:	$s''(t) = v'(t) = a(t)$

Now, if we consider working backward, we can get integration rules as well:

$$\int a(t)dt = v(t) + C$$

$$\int v(t)dt = s(t) + C$$

Now, we can get the position function from the acceleration function with a few initial conditions.

Acceleration due to gravity is $9.8\ \mathrm{m/s^2}$. This info gives us the acceleration function, which when integrated gives us the velocity function, which when integrated gives us the position function.

$$a(t) = -9.8$$

$$\int a(t)dt = \int -9.8dt = -9.8t + C = v(t)$$

Given that the initial velocity, $v(0)$, is 26, we can find C.

$$v(0) = -9.8(0) + C = 26$$

$$C = 26$$

$$v(t) = -9.8t + 26$$

Integrate again.

$$\int v(t)dt = \int (-9.8t + 26)dt = -4.9t^2 + 26t + C = s(t)$$

Given that the initial position, $s(0)$, is 15, we can find C for this function.

$$s(0) = -4.9(0)^2 + 26(0) + C = 15$$

$$C = 15$$

$$s(t) = -4.9t^2 + 26t + 15$$

position function