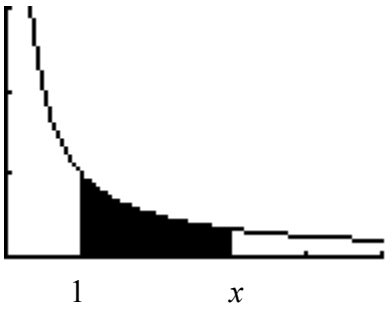
Chapter 5 Review Answers

1. Define $\ln x$ and illustrate the definition.

$$\ln x = \int_1^x \frac{1}{t}\,dt$$



In questions 2–5, complete the statement.

2. $\ln e =$ 1

3. $\ln 1 =$ 0

4. $e^{\ln x} =$ x

5. $\log_a a^x =$ x

6. Write $\frac{1}{3}\ln x + 5\ln y - 2\ln z$ as a single logarithm.

$$\ln \frac{x^{1/3}y^5}{z^2}$$

7. Expand $\ln\left(\frac{3x^2}{\sqrt{x-2}}\right)$.

$$\ln 3 + 2\ln x - \frac{1}{2}\ln(x-2)$$

In questions 8–10, solve for *x*.

8. $\log_2 \frac{1}{16} = x$

$$2^x = \frac{1}{16}$$
$$x = -4$$

9. $7e^{2x} = 42$

$$e^{2x} = 6$$
$$2x = \ln 6$$
$$x = \frac{\ln 6}{2}$$

10. $\ln x^5 = 50$

$$5\ln x = 50$$
$$\ln x = 10$$
$$x = e^{10}$$

In questions 11–19, find the derivative of each function.

11. $y = (\ln x)^5$

$$y' = 5(\ln x)^4 \cdot \frac{1}{x}$$
$$y' = \frac{5(\ln x)^4}{x}$$

12. $y = \ln\sqrt{x^3 + 7}$

$$y = \frac{1}{2}\ln(x^3 + 7)$$
$$y' = \frac{1}{2} \cdot \frac{1}{x^3 + 7} \cdot 3x^2$$
$$y' = \frac{3x^2}{2(x^3 + 7)}$$

13. $f(x) = e^{6x^5}$

$$f'(x) = e^{6x^5} \cdot 30x^4$$
$$f'(x) = 30x^4 e^{6x^5}$$

14. $y = \ln e^{x^3}$

$$y' = \frac{1}{e^{x^3}} \cdot e^{x^3} \cdot 3x^2$$
$$y' = 3x^2$$

15. $y = 10^x$

$$y' = 10^x \ln 10$$

16. $y = \log_2(16x^4)$

$$y = \log_2 16 + 4\log_2 x$$
$$y = 4 + 4\log_2 x$$
$$y' = 4 \cdot \frac{1}{x\ln 2}$$
$$y' = \frac{4}{x\ln 2}$$

17. $f(x) = 4^x \cdot x^4$

$$f'(x) = 4^x(4x^3) + x^4(4^x \ln 4)$$

18. $y = \log_2 \frac{2x}{x+5}$

$$y' = \frac{1}{\left(\frac{2x}{x+5}\right)\ln 2} \cdot \frac{(x+5)2 - 2x(1)}{(x+5)^2}$$
$$y' = \frac{x+5}{2x\ln 2} \cdot \frac{10}{(x+5)^2}$$
$$y' = \frac{5}{x(x+5)(\ln 2)}$$

19. $y = x^{5x}$

$$\ln y = \ln(x^{5x})$$
$$\ln y = 5x \ln x$$
$$\frac{1}{y} \cdot y' = 5x \cdot \frac{1}{x} + \ln x \cdot 5$$
$$y' = y(5 + 5\ln x)$$
$$y' = x^{5x}(5 + 5\ln x)$$

20. Find $\frac{dy}{dx}$ for $y = x \cdot e^y + 6x$.

$$\frac{dy}{dx} = x \cdot e^y \frac{dy}{dx} + e^y + 6$$
$$\frac{dy}{dx} - xe^y \frac{dy}{dx} = e^y + 6$$
$$\frac{dy}{dx}(1 - xe^y) = e^y + 6$$
$$\frac{dy}{dx} = \frac{e^y + 6}{1 - xe^y}$$

In questions 21–30, find each integral.

21. $\int \frac{1}{4x} dx$

$\frac{1}{4} \ln |x| + C$

22. $\int \frac{x^3}{3 - x^4} dx$

$-\frac{1}{4} \ln |3x - x^4| + C$

23. $\int \cot 7x \, dx$

$\frac{1}{7} \ln |\sin 7x| + C$

24. $\int_1^e \frac{(\ln x)^4}{x} dx$

$\int_0^1 u^4 du = \frac{1}{5} u^5 \Big]_0^1 = \frac{1}{5}$

25. $\int_3^4 e^{6-x}\,dx$

$$-\int_3^2 e^u\,du = -e^u\Big]_2^3 = -e^2 + e^3$$

26. $\int x \cdot e^{3x^2}\,dx$

$$\frac{1}{6}\int e^u\,du = \frac{1}{6}e^{3x^2} + C$$

27. $\int \frac{e^x}{5+e^x}\,dx$

$$\int \frac{1}{u}\,du = \ln|5+e^x| + C$$

28. $\int 7^{-x}\,dx$

$$\int \left(\frac{1}{7}\right)^x dx = \frac{1}{\ln\left|\frac{1}{7}\right|}\cdot\left(\frac{1}{7}\right)^x + C$$

29. $\int_1^{\sqrt[3]{2}} x^3 \cdot 5^{x^4}\,dx$

$$\frac{1}{4}\int_1^{\left(\sqrt[3]{2}\right)^4} 5^u\,du = \frac{1}{4}\left[\frac{1}{\ln 5}\cdot 5^u\right]_1^{\left(\sqrt[3]{2}\right)^4}$$

$$= \frac{5^{\left(\sqrt[3]{2}\right)^4} - 5}{4\ln 5}$$

30. $\int_0^{\pi/2} \cos x \cdot e^{\sin x}\,dx$

$$\int_0^1 e^u\,du = e^u\Big]_0^1$$

$$= e - 1$$

31. Prove that $\int \tan x\,dx = \ln|\sec x| + C$.

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$$= -\int \frac{1}{u}\,du$$

$$= -\ln|\cos x| + C$$

$$= -\ln|\sec x| + C$$

32. Solve the differential equation $\frac{dy}{dt} = ky$ for y. Show your steps.

$$\frac{dy}{dt} = ky$$

$$\frac{1}{y}\,dy = k\,dt$$

$$\ln y = kt + C$$

$$e^{\ln y} = e^{kt+C}$$

$$y = e^{kt} \cdot e^{C}$$

$$y = Ce^{kt}$$

33. During lunch, you eat a french fry from a sick friend's plate and ingest 40 bacteria. Six hours later, the bacteria population has grown exponentially to 200. Find the value of k and write the appropriate equation to find how many bacteria will be present in 48 hours (do not solve).

$$200 = 40e^{6k}$$

$$5 = e^{6k}$$

$$\ln 5 = 6k$$

$$k = \frac{\ln 5}{6}$$

$$y = 40e^{\frac{\ln 5}{6}(48)}$$