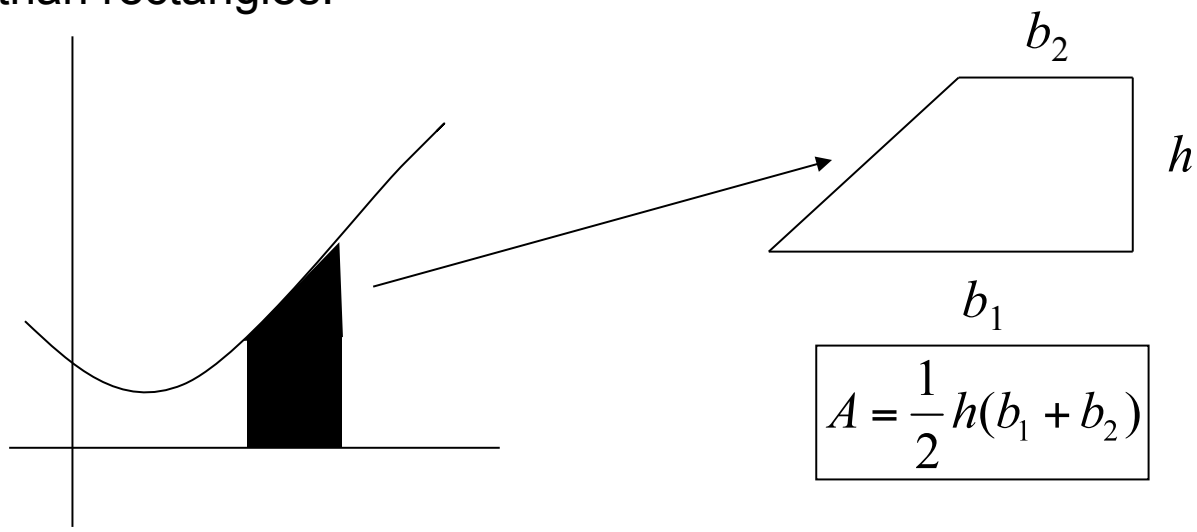
Trapezoidal Rule

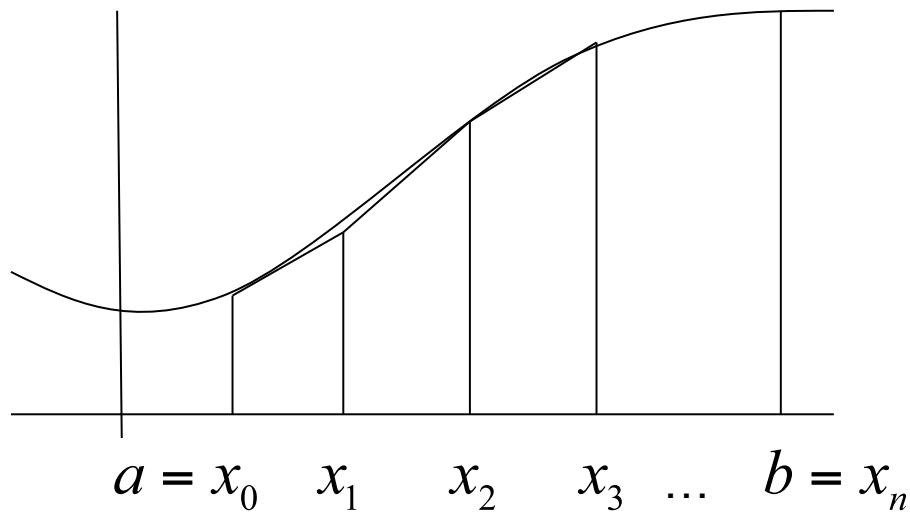
This is a new way to approximate area and it is more accurate than rectangles.



$$\boxed{A = \frac{1}{2}h(b_1 + b_2)}$$

In general,

$$h = \Delta x = \frac{b-a}{n}$$



$$A = \frac{1}{2}\Delta x[f(x_0) + f(x_1)] + \frac{1}{2}\Delta x[f(x_1) + f(x_2)] + \cdots + \frac{1}{2}\Delta x[f(x_{n-1}) + f(x_n)]$$

$$A = \frac{1}{2}\Delta x[f(x_0) + f(x_1) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + f(x_n)]$$

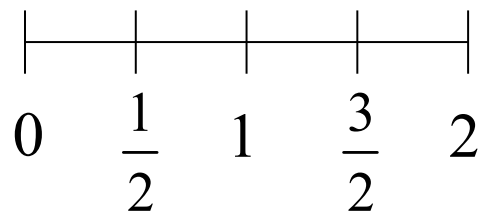
$$A = \frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

This simplification leads us to the Trapezoidal Rule.

Trapezoidal Rule

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}[f(x_0)+2f(x_1)+\cdots+2f(x_{n-1})+f(x_n)]$$

Example: $\int_0^2 x^2 dx$

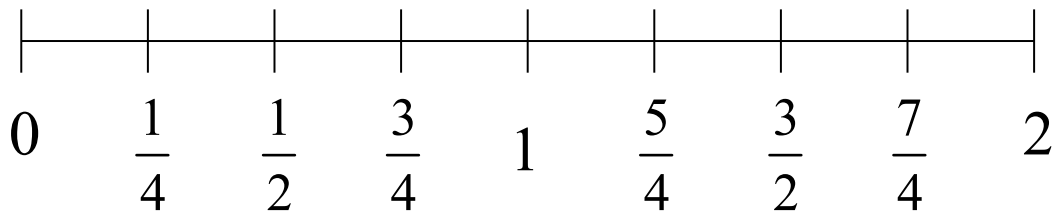


Use the Trapezoidal Rule with $n = 4$. (Take the interval from 0 to 2 and break it up into 4 parts.)

$$A = \frac{2-0}{8}\left[f(0)+2f\left(\frac{1}{2}\right)+2f(1)+2f\left(\frac{3}{2}\right)+f(2)\right]$$

$$A = \frac{1}{4}\left(0+\frac{1}{2}+2+\frac{9}{2}+4\right) = \frac{11}{4}$$

Now try it with $n = 8$.



$$A = \frac{2-0}{16}\left[f(0) + 2f\left(\frac{1}{4}\right) + 2f\left(\frac{1}{2}\right) + 2f\left(\frac{3}{4}\right) + 2f(1) + 2f\left(\frac{5}{4}\right) + 2f\left(\frac{3}{2}\right) + 2f\left(\frac{7}{4}\right) + f(2)\right]$$

$$A = \frac{1}{8}\left(0 + \frac{1}{8} + \frac{1}{2} + \frac{9}{8} + 2 + \frac{25}{8} + \frac{9}{2} + \frac{49}{8} + 4\right) = \frac{43}{16}$$