# Antiderivatives and Integration

Consider the following function:

$$f(x) = 4x^3$$

That function is the derivative of what?

$$x^4$$

Keep in mind this is ***an*** antiderivative, not ***the*** antiderivative.

**Definition:** $F$ is an antiderivative of $f$ if $F'(x) = f(x)$.

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| $F$ (capital $f$) is the variable we use to notate the antiderivative. |

**Theorem:** Two antiderivatives of the same function can differ at most by a constant.

Let $f(x) = 4x^3$.

$F(x) = x^4 + C$ represents a family of all possible antiderivatives of $f(x) = 4x^3$.

Here is an alternative notation:

Recall that

$$\frac{dy}{dx} = f'(x).$$

Consider the following.

$$\frac{dy}{dx} = 4x^3 \quad \text{(differential equation)}$$

$$y = x^4 + C$$

# Integral Notation

$$\int 4x^3 dx = x^4 + C$$

integral

integrand

variable of integration

This means “the antiderivative of $f$ with respect to $x$” or “the indefinite integral of $f$ with respect to $x$.”

Going back to differentials, we write the following:

$$y = x^4 + C$$

$$\frac{dy}{dx} = 4x^3$$

$$dy = 4x^3 dx$$

In general,

$$\int f(x)dx = F(x) + C.$$

Recall $F' = f$ ($F'$ is the derivative of the integral), so

$$\int F'(x)dx = F(x) + C$$

and

$$\frac{d}{dx}\left[\int f(x)dx\right] = f(x).$$

Integration and differentiation are inverses.

$$\int 2xdx = x^2 + C$$

$$\int 2dx = 2x + C$$

$$\int \sin xdx = -\cos x + C$$

$$\int \cos xdx = \sin x + C$$

Every derivative rule leads to an integration rule.

$$\int kdx = C$$

$$\int kf(x)dx = k\int f(x)dx$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad n \neq -1$$

Let's try a few examples.

1. $\int (x^2 + 8x + 2)\,dx = \boxed{\frac{1}{3}x^3 + 4x^2 + 2x + C}$

Don't forget "$+\ C$."

2. $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \boxed{-x^{-1} + C \quad \text{or} \quad -\frac{1}{x} + C}$

3. $\int 3\sqrt{x}\,dx = 3\int x^{1/2}\,dx = 3\left(\frac{2}{3}x^{3/2} + C\right) = \boxed{2x^{3/2} + C}$

We don't need to write $3C$, because $C$ doesn't need to be defined. We just need to know that it is some constant.