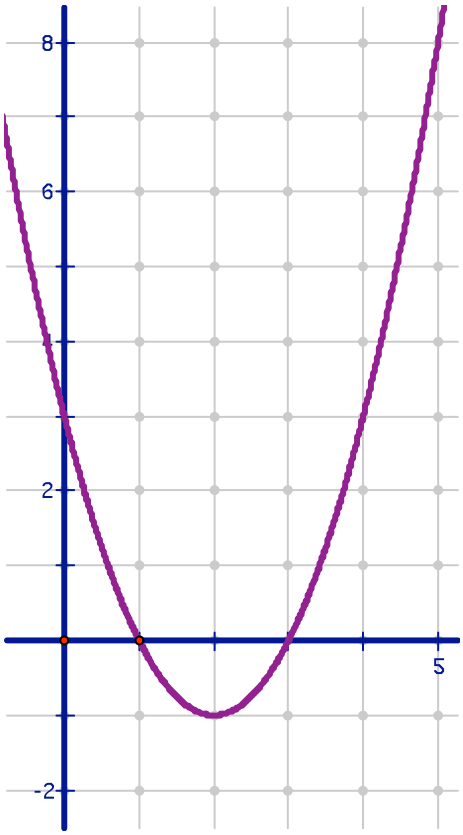
Area Under the Curve
When the Graph Crosses the x-axis

Let's find the area between $f(x) = x^2 - 4x + 3$ and the x-axis on $[0, 5]$.

Generally, we just integrate the function from 0 to 5.

$$\int_0^5 (x^2 - 4x + 3)\,dx = \left[\frac{1}{3}x^3 - 2x^2 + 3x\right]_0^5 = \frac{20}{3}$$

So, this is the area. But, let's take a look at the graph.



Notice the graph goes below the x-axis between $x = 1$ and $x = 3$. Integrating would calculate this as negative, but the area is not negative, so we need to take that into consideration when integrating.

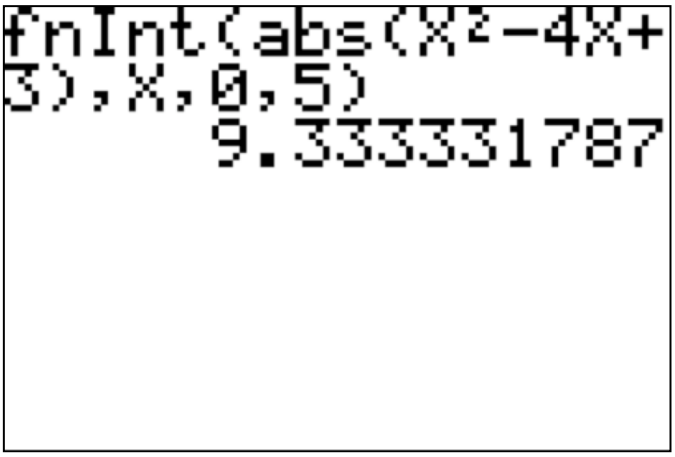
The actual area would be calculated in the following way:

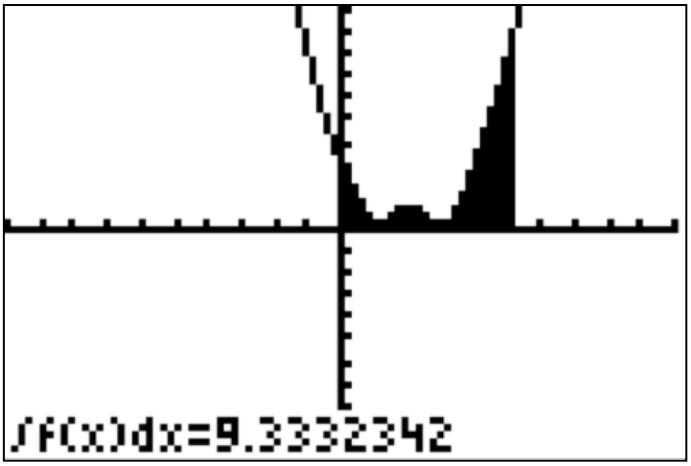
$$\int_0^1 (x^2 - 4x + 3)\,dx - \int_1^3 (x^2 - 4x + 3)\,dx + \int_3^5 (x^2 - 4x + 3)\,dx$$

$$\frac{4}{3} \quad - \quad \left(-\frac{4}{3}\right) \quad + \quad \frac{20}{3}$$

$$\frac{28}{3}$$

We can also take care of this by using the absolute value with the calculator.





Notice the calculator does not give the exact answer, probably due to the algorithm used to do the calculations or the amount of memory your calculator has available. The answer is correct to three decimal places, which is the standard used by the college board on the AP Calculus AB exam.

Review

Definition of Definite Integral

$$\int_a^b f(x)\,dx = \lim_{\|\Delta x\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

Fundamental Theorem of Calculus (FTC)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Average Value of a Function on an Interval

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$