Definite Integrals with u Substitution

The thing we really have to notice in this section is when we change the function from x to u, we also have to change the upper and lower bounds. To do that, we just use the equation we have for u.

Example: Find $\int_1^3 6x(3x^2+1)^2\,dx$.

Let $u = 3x^2+1$.

$du = 6x\,dx$

Now we convert the upper and lower bounds from x to u.

$$x = 1 \Rightarrow u = 3(1)^2+1 = 4$$

$$x = 3 \Rightarrow u = 3(3)^2+1 = 28$$

Now we can rewrite the definite integral:

$$\int_4^{28} u^2\,du = \frac{1}{3}u^3\Big]_4^{28} = \frac{1}{3}(28^3-4^3) = 7296$$

Note that since we changed our bounds, we do not have to write the integral in terms of x.

Example: Find $\int_{\pi/6}^{\pi/2} \sin^2\theta \cos\theta d\theta$.

Let $u = \sin\theta$.

$du = \cos\theta d\theta$

Now we convert the upper and lower bounds from x to u.

$$x = \frac{\pi}{6} \Rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$x = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$$

Now we can rewrite the definite integral:

$$\int_{1/2}^{1} u^2 du = \frac{1}{3}u^3\Big]_{1/2}^{1} = \frac{1}{3}\left[1^3 - \left(\frac{1}{2}\right)^3\right] = \frac{7}{24}$$

Example: Find $\int_{-1}^{1} x(x^2+1)^3 dx$.

Let $u = x^2 + 1$.

$du = 2x\,dx$

Now we convert the upper and lower bounds from x to u.

$$x = -1 \Rightarrow u = (-1)^2 + 1 = 2$$

$$x = 1 \Rightarrow u = (1)^2 + 1 = 2$$

Now we can rewrite the definite integral:

$$\frac{1}{2}\int_{2}^{2} u^3 du = 0$$

This rewrite makes it simple enough to use one of our integral rules.