Particle Movement

A particle moves left or right, along the x-axis.

- If the velocity is positive, the particle is moving to the right.
- If the velocity is negative, the particle is moving to the left.

Displacement is the difference between where the particle started and where it ended.

Distance is the total distance the particle traveled.

Example: A particle moves along the x-axis with velocity $v(t) = t^2 - 7t + 10$ for $0 \leq t \leq 6$ and initial condition of position 4 at time zero.

a. Find the position function.
b. Find the displacement.
c. Find the total distance traveled.

a. Finding the position function

$$s(t) = \int v(t)dt$$

$$s(t) = \int (t^2 - 7t + 10)dt$$

$$s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + C$$

Position is 4 when the time is zero. We can solve for C.

$$s(0) = \frac{1}{3}(0)^3 - \frac{7}{2}(0)^2 + 10(0) + C = 4$$

$$C = 4$$

$$s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + 4$$

b. Finding the displacement

We can do this two ways. For the first way, we will find the position at 0 seconds and the position at 6 seconds, then subtract.

$$s(0) = \frac{1}{3}(0)^3 - \frac{7}{2}(0)^2 + 10(0) + 4 = 4$$

$$s(6) = \frac{1}{3}(6)^3 - \frac{7}{2}(6)^2 + 10(6) + 4 = 10$$

$$s(6) - s(0) = 10 - 4 = 6$$

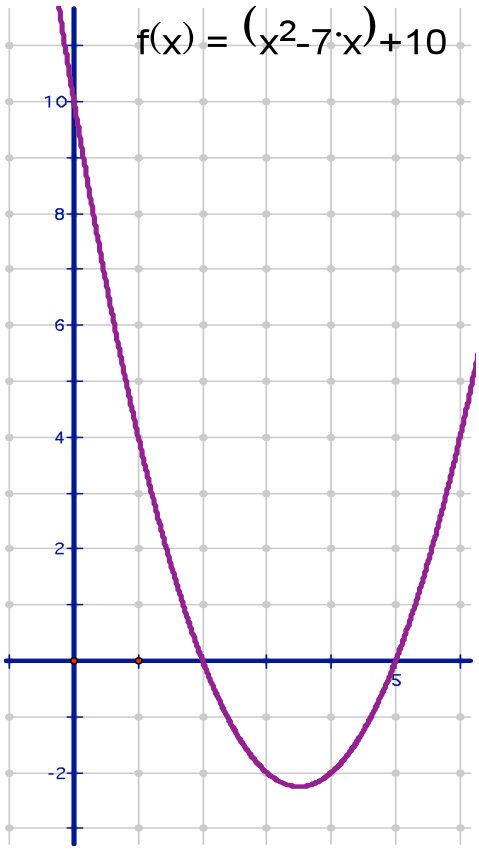
$$\boxed{\text{Positive, since it moved from } 4 \text{ to } 10.}$$

For the second way, we will integrate the velocity function from 0 to 6.

$$\begin{aligned}\int_0^6 (t^2 - 7t + 10)\,dt &= \left[\frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t\right]_0^6 \\ &= \frac{1}{3}(6)^3 - \frac{7}{2}(6)^2 + 10(6) - 0 \\ &= 6\end{aligned}$$

c. Finding the total distance traveled

To find the total distance traveled, we have to consider the function moving back toward the origin. Consider the graph.



From the graph, it looks like the particle is moving away from the origin from 0 to 2, then back towards the origin from 2 to 5, then away again from 5 to 6. If we find the position at 0, 2, 5, and 6, we can find the total distance.

$$s(0) = \frac{1}{3}(0)^3 - \frac{7}{2}(0)^2 + 10(0) + 4 = 4$$

$$s(2) = \frac{1}{3}(2)^3 - \frac{7}{2}(2)^2 + 10(2) + 4 = 12\tfrac{2}{3}$$

$$s(5) = \frac{1}{3}(5)^3 - \frac{7}{2}(5)^2 + 10(5) + 4 = 8\tfrac{1}{6}$$

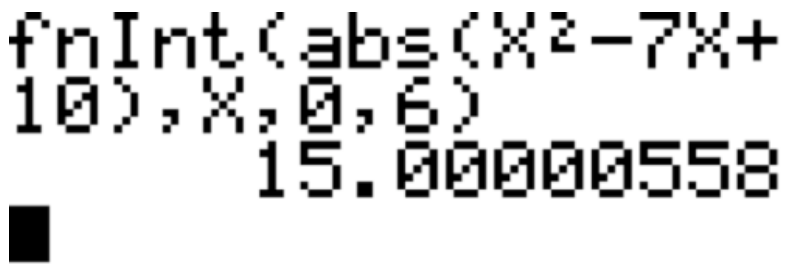
$$s(6) = \frac{1}{3}(6)^3 - \frac{7}{2}(6)^2 + 10(6) + 4 = 10$$

$8\tfrac{2}{3}$

$4\tfrac{1}{2}$

$1\tfrac{5}{6}$

$$\boxed{\text{Total Distance} = 15}$$

The second way is to integrate the absolute value of the function from 0 to 6 on the graphing calculator.

$$\int_0^6 \left| t^2 - 7t + 10 \right| dt = 15$$