Practice Final Exam

1. Which of the following statements is true of $f(x) = -x^3 + 18x^2 - 105x + 198$?

   a. f is increasing on $(-\infty, 5)$.
   b. f is decreasing on $(6, \infty)$.
   c. f is increasing on $(5, 7)$.
   d. f is decreasing on $(5, 7)$.
   e. none of these

2. Given that $f(x) = -x^2 + 18x - 78$ has a relative maximum at $x = 9$, choose the correct statement.

   a. f' is positive on the interval $(9, \infty)$.
   b. f' is negative on the interval $(9, \infty)$.
   c. f' is negative on the interval $(-\infty, 9)$.
   d. f' is positive on the interval $(-\infty, \infty)$.
   e. none of these

3. Find all points of inflection of the graph of the function $f(x) = x^4 + x^3$.

   a. $(0, 0)$ and $\left(-\frac{1}{2}, -\frac{1}{16}\right)$
   b. $(0, 0)$ and $\left(-\frac{3}{4}, -\frac{27}{256}\right)$
   c. $(0, 0)$
   d. $\left(-\frac{1}{2}, -\frac{1}{16}\right)$
   e. none of these

4. Let $f''(x) = 3x^2 - 4$ and let $f(x)$ have critical numbers -2, 0, and 2. Use the Second Derivative Test to determine which critical numbers, if any, give a relative maximum.

   a. 2
   b. 0
   c. -2 and 2
   d. -2
   e. none of these

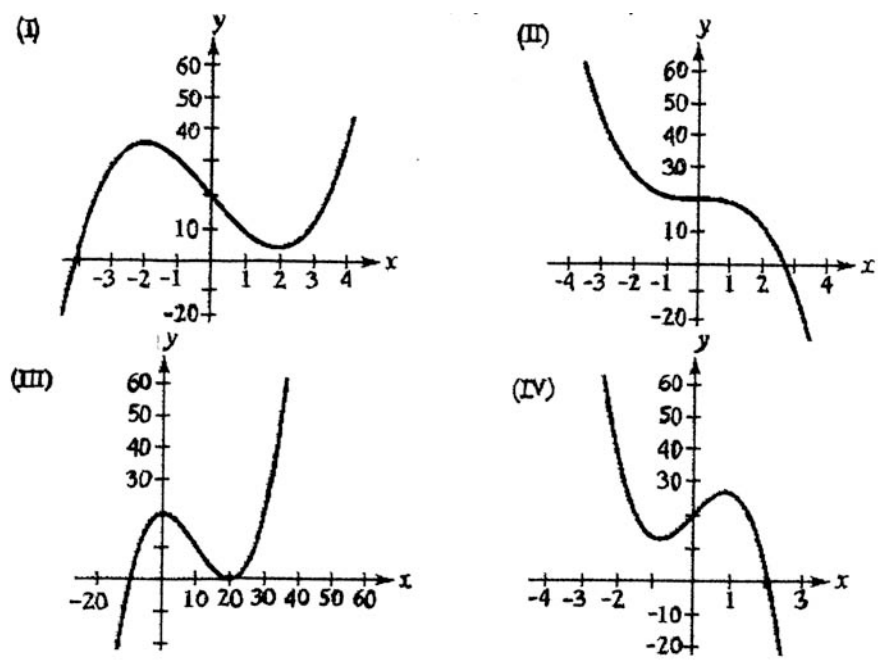
5. Find the horizontal asymptote for $f(x) = \dfrac{7}{x-4}$.

   a. $x = 4$
   b. $y = 7$
   c. $x = 0$
   d. $y = 0$
   e. none of these

6. Find the limit: $\lim\limits_{x \to \infty} \dfrac{a - bx^4}{cx^4 + x^2}$.

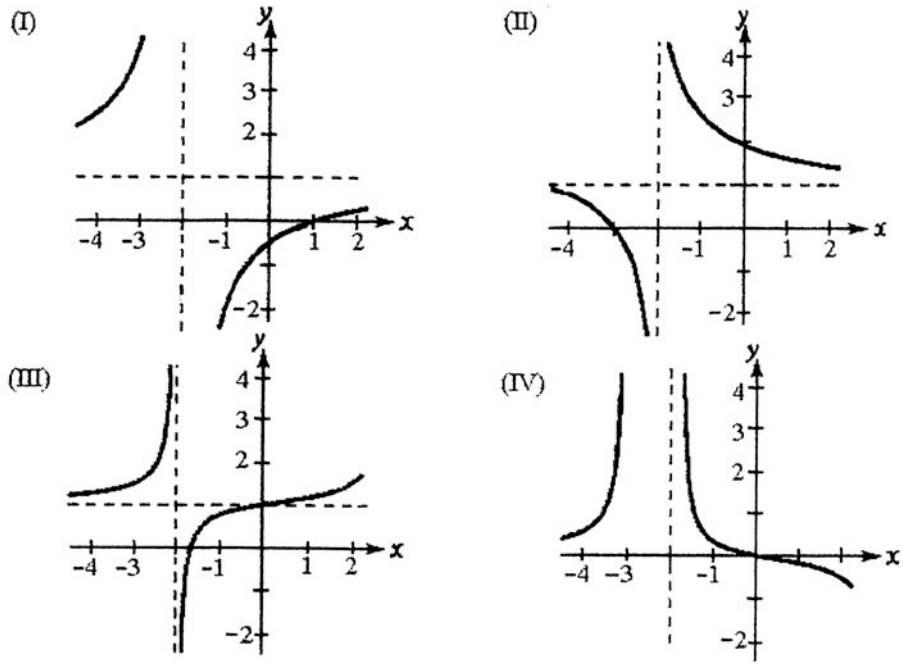
   a. $\dfrac{a}{c}$
   b. 0
   c. ∞
   d. $-\dfrac{b}{c}$
   e. none of these

7. Which of the following is the correct sketch of the graph of the function $y = x^3 - 12x + 20$?



   a. IV
   b. III
   c. II
   d. I
   e. none of these

8. Which of the following is the correct sketch of the graph of the function $f(x) = \dfrac{x-1}{x+2}$?



a. III
b. II
c. IV
d. I
e. none of these

9. A farmer has 160 feet of fencing to enclose two adjacent rectangular pigpens. What dimensions should be used so that the enclosed area will be a maximum?

a. $4\sqrt{15}$ ft by $\frac{8}{5}\sqrt{15}$ ft
b. 20 ft by $\frac{80}{3}$ ft
c. 40 ft by $\frac{80}{3}$ ft
d. 40 ft by 40 ft
e. none of these

10. Use differentials to approximate $\sqrt{4.9}$.

a. 2.225
b. 2.450
c. 2.214
d. 2.250
e. none of these

11. Find the integral: $\int (2x^4 + 3x^2 - 2x)dx$.

a. $\frac{2}{5}x^5 + x^3 - x^2 + C$
b. $8x^3 + 6x - 2 + C$
c. $2x^5 + 5x^3 - 5x^2 + C$
d. $\frac{3}{8}x^8 + C$
e. none of these

12. Find the integral: $\int \frac{1}{t^6} dt$.

a. $-\frac{1}{5t^5} + C$
b. $-\frac{6}{t^7} + C$
c. $\frac{7}{t^7} + C$
d. $\frac{1}{6t^7} + C$
e. none of these

13. Find the integral: $\int \sqrt[5]{x^2}\, dx$.

a. $-\frac{1}{4x^8} + C$
b. $-\frac{1}{9x^9} + C$
c. $\frac{5}{7}x^{7/5} + C$
d. $\sqrt[5]{\frac{x^3}{3}} + C$
e. none of these

14. Find $y = f(x)$ if $f''(x) = x^2$, $f'(0) = 7$, and $f(0) = 2$.

a. $\frac{1}{12}x^4 + 7x + 2$
b. $x^2 + 7x + 2$
c. $x^2 + 9$
d. $x^4 + 84x + 24$
e. none of these

15. Choose the correct statement given that $\int_0^7 f(x)dx = 8$ and $\int_1^7 f(x)dx = -3$.

a. $\int_7^1 f(x)dx = -3$
b. $\int_0^1 f(x)dx = 11$
c. $\int_1^0 f(x)dx = 11$
d. $\int_0^1 f(x)dx = 5$
e. none of these

16. Use the Fundamental Theorem of Calculus to evaluate $\int_1^2 \frac{1}{x^2}\,dx$.

a. $-\frac{3}{4}$
b. $-\frac{1}{x} + C$
c. $-\frac{3}{2}$
d. $\frac{1}{2}$
e. none of these

17. Use the Fundamental Theorem of Calculus to evaluate $\int_{\pi/4}^{\pi/3} \sec^2 x\,dx$.

18. Find the average value of $f(x) = 3x^2 - 2$ on the interval [0, 2].

a. 1
b. 2
c. 6
d. 5
e. none of these

19. Find the integral: $\int x(x^2-1)^4\,dx$.

a. $\frac{1}{10}(x^2 - 1)^5 + C$
b. $\frac{1}{5}(x^3 - x)^5 + C$
c. $\frac{1}{5}(x^2 - 1)^5 + C$
d. $\frac{1}{10}x^2(x^2 - 1)^5$
e. none of these

20. Find the integral: $\int x \sec^2 x^2 dx$.

a. $\frac{1}{6}x^3 \sec^3 x^2 + C$
b. $\frac{1}{2}x^2 \tan x^2 + C$
c. $\tan x^3 + C$
d. $\frac{1}{2} \tan x^2 + C$
e. none of these

21. Choose the expression equivalent to $\ln \frac{8x^2}{3y}$.

a. $\ln 8 - \ln 3 + 2 \ln x - \ln y$
b. $\ln(8x^2) + \ln(3y)$
c. $2 \ln(8x) - \ln(3y)$
d. $\frac{\ln 8 + \ln x^2}{\ln 3 + \ln y}$
e. none of these

22. Find the derivative: $f(x) = \ln \frac{x^2\sqrt{4x+1}}{(x^3+5)^3}$.

a. $\frac{2}{x} - \frac{2}{4x+1} - \frac{9x^2}{x^3+5}$
b. $\frac{x}{9x^2(x^3+5)^2\sqrt{4x+1}}$
c. $\frac{2}{x} + \frac{2}{4x+1} - \frac{9x^2}{x^3+5}$
d. $\frac{2}{x} + \frac{1}{2(4x+1)} - \frac{3}{x^3+5}$
e. none of these

23. Evaluate the integral: $\int_1^{5e} \frac{1}{x} dx$.

a. $1 + \ln 5$
b. ∞
c. 0
d. $\frac{1}{5e} - 1$
e. none of these

24. Find the integral: $\int \frac{2x+1}{x+1}\,dx$.

a. $2x + C$
b. $2x - \ln|x+1| + C$
c. $\frac{2x^2+2x}{x^2+2x} + C$
d. $x^2 + \ln|x+1| + C$
e. none of these

25. Given $f(x) = 3x^3 - 1$, find $f^{-1}(x)$.

a. $\sqrt[3]{\frac{x+1}{3}}$
b. $\frac{1}{3x^3-1}$
c. $3x^{-1} - 1$
d. $3(x+1)$
e. none of these

26. Differentiate with respect to x: $y = e^{7-(5/x)}$.

a. $\frac{dy}{dx} = e^{5/x^2}$
b. $\frac{dy}{dx} = -e^{7-(5/x)}$
c. $\frac{dy}{dx} = \frac{5}{x^2}e^{7-(5/x)}$
d. $\frac{dy}{dx} = e^{7-(5/x)}$
e. none of these

27. Find $\frac{dy}{dx}$ if $ye^x - x = y^2$.

28. Find the indefinite integral: $\int \frac{1}{x^2 e^{2/x}}\,dx$.

a. $\frac{1}{2}e^{-2/x} + C$
b. $\frac{1}{2}xe^{-2/x} + C$
c. $\frac{1}{2}xe^{2/x} + C$
d. $\frac{1}{2}e^{2/x} + C$
e. none of these

29. Find $\frac{dy}{dx}$ if $y = 3^x x^3$.

a. $9x^2$
b. $3^{x-1}x^2[9 + x^2]$
c. $3^x x^2[3 + (\ln 3)x]$
d. $2x^2 3^{x-1}$
e. none of these

30. Find the integral: $\int 3x \cdot 5^{x^2}\,dx$.

a. $\frac{3 \cdot 5^{x^2}}{2\ln 5} + C$
b. $\frac{2 \cdot 5^{x^2}}{3\ln 5} + C$
c. $3 \cdot 5^{x^2}[2x^2 \ln 5 + 1]$
d. $\frac{3}{2} \cdot 5^{x^2} + C$
e. none of these